IA waves in a plasma with vortex-like negative ions

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This investigated has been made to study the basic features of the nonlinear waves, associated with positive ion dynamics in a plasma containing Boltzmann electrons,and vortex-like negative ions using the reductive perturbation method. It has been observed that the basic features of the nonlinear solitary and shock waves have been significantly modified by the trapping parameter for vortex-like distribution of negative ions. The outcomes of our this investigation will help to verify those results come from plasma space and laboratory experiments .

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I. INTRODUCTION

The electro-negative plasmas, [1, 2] which are observed in both space and laboratory devices, are provided with a significant amount of negative ions whose contribution cannot be neglected in any ways. The propagation of nonlinear waves in such a plasma system has been studied by a number of authors [2–4]. They have considered considered ion-acoustic shock waves (associated with the dynamics of negative ions) in a multi-ion dusty plasma containing electrons, light positive ions, heavy negative ions, and extremely massive charge fluctuating stationary dust. They have also considered negative (positive) dust charging currents [5–7], where negative ions are not in Blotzmann equilibrium and current fluctuation associated with them has been neglected.

On the other hand, it has been predicted by a number of authors that negative ions in such electro-negative plasmas are in Boltzmann equilibrium. This prediction has been conclusively verified by a recent laboratory experiment. Recently, a number of authors [8, 9] have considered dusty electro-negative plasma system containing Boltzmann electrons, Boltzmann negative ions [10, 11], cold mobile positive ions.

Very recently, a theoretical investigation have been made on the nonlinear propagation of waves in dusty electro-negative plasma with Boltzmann electrons, Boltzmann negative ions, cold mobile positive ions, and charge fluctuating stationary dust [12]. However, due to wave-particle interactions, negative ions may not be isothermal (Boltzmann), but they follow other distributions, particularly vortex-like (trapped) [13] distribution, which has very important role in basic features of nonlinear electrostatic structures in many space [14, 15] and laboratory [16, 17] plasma.

Therefore, in our present work, we consider a simple plasma system containing Boltzmann electrons, trapped negative ions and mobile positive ions to study the basic properties (viz. amplitude, width etc.) of nonlinear ion-acoustic (IA) waves.

II. GOVERNING EQUATIONS

The nonlinear dynamics of our considered plasma system is governed by the following equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\sigma_i}{n_i}\frac{\partial n_i}{\partial x} + \eta \frac{\partial u_i^2}{\partial x^2}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = I \exp(\phi) + \mu_n \left[1 + \sigma_n \phi\right]^{3/2} - \frac{4}{3\sqrt{\pi}}\left(1-\beta_n\right)\left(\sigma_n \phi + \frac{1}{2}\left(\sigma_n \phi\right)^2\right] - n_i, \tag{3}$$

$$I = I_e + I_n + I_i, \tag{4}$$

where $\lambda_{Dm} = (T_e/4\pi n_{i0} e^2)^{1/2}$ is the long wavelength, T_e is the electron thermal energy and e is the magnitude of the electron charge, n_i is the positive ion number density normalized by its equilibrium value n_{i0}, u_i is the speed of positive ion fluid normalized by the ion-acoustic speed $C_i = (T_e/m_i)^{1/2}$, ϕ is the electrostatic wave potential normalized by T_e/e. Here, β_n is the trapping parameter determined by the ratio of free and trapped negative ion temperatures. I_e I_n, and I_i are respectively, electron, negative ion, and positive ion current, x is the space variable normalized by λ_{Dm} and t is the time variable normalized by the ion plasma period $\tau_i^{-1} = (m_i/4\pi n_{i0} e^2)^{1/2}$, $\sigma_i = T_i/T_e$, $\alpha = n_{n0}/n_{e0}$, $\sigma_n = T_e/T_n$, n_{e0}, n_{n0}, and n_{d0} are, respectively, electron, negative ion, and dust number densities at equilibrium, T_i (T_n) is the positive (negative) ion thermal energy, and m_i is the ion mass. The normalized electron, negative ion, and positive ion currents (I_e, I_n, and I_i) are given by

$$I_e = I_{e0} \exp(\phi - \gamma_e), \tag{5}$$

$$I_n = I_{n0} \exp(\sigma_n \phi - \gamma_n), \tag{6}$$

$$I_i = I_{n0}{'} \exp \beta_n(\alpha_1) + I_{n0}{''}[1 + \gamma_1(\alpha_1 + \sigma_n \phi)], \tag{7}$$

where

$$I_{e0} = -2\sqrt{2\pi} r_d^2 \lambda_{dm} n_{e0} \sqrt{\frac{m_i}{m_e}} \exp(-\gamma_e),$$

$$I_{n0} = -e\pi {r_d}^2 n_{n0}\sqrt{\frac{8T_{nf}}{\pi m_n}}\,[\exp(\beta_n Q_{do})$$
$$+(1-{\beta_n}^{-1})(Q_{do}+1+{\beta_n}^{-1})]\,,$$
$$I_{n0}{}' = I_{n0}\frac{\exp(\beta_n Q_{do})}{(\exp\beta_n Q_{do})+(1-{\beta_n}^{-1})(Q_{do}+1+{\beta_n}^{-1})},$$
$$I_{n0}{}'' = I_{n0}\frac{(1-{\beta_n}^{-1})(Q_{do}+1+{\beta_n}^{-1})}{\exp(\beta_n Q_{do})(1-{\beta_n}^{-1})(Q_{do}+1+{\beta_n}^{-1})},$$
$$I_{i0} = I_{i0}^0(1+\gamma_i),$$
$$I_{i0}^0 = \sqrt{2}\pi r_d^2\lambda_{dm}n_{i0}\sqrt{\frac{T_k}{T_e}},$$

$\gamma_e = e^2/r_d T_e$, $\gamma_n = e^2/r_d T_{nf}$, $\gamma_i = e^2/r_d m_i u_{i0}^2$, and u_{i0} is the positive ion streaming speed which is assumed to be much larger than its thermal speed. The current equation for negative ion (viz. I_{n0}) follows the vortex-like distribution.

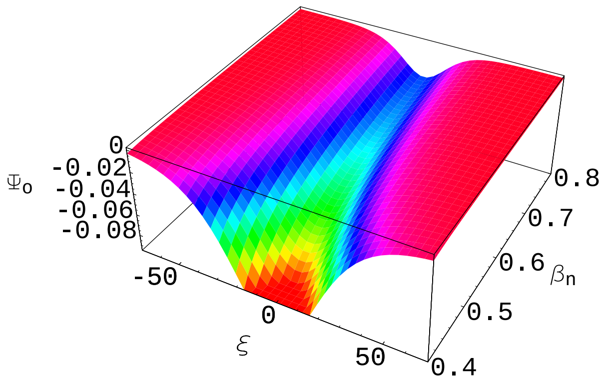

FIG. 1: (Color online) Showing the variation of Φ_0 with β_n for solitary waves when $U_0 = 0.1$, $\sigma_i = 0.4$, and $\sigma_n = 0.6$.

III. DERIVATION OF K-DV EQUATION

To derive a dynamical equation for the DIA solitary waves from our basic equations (1)-(4), we again employ the reductive perturbation technique with another stretched coordinates [18, 19]

$$\xi = \epsilon^{1/2}(x - V_p t), \tag{8}$$
$$\tau = \epsilon^{3/2}t. \tag{9}$$

where ϵ is a small parameter ($0 < \epsilon < 1$) measuring the weakness of the dispersion, and V_p (normalized by C_i) is the phase speed of the perturbation mode. Now, we expand n_i, u_i and ϕ in power series of ϵ, viz.

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^{3/2}n_i^{(2)} + \cdots, \tag{10}$$
$$u_i = \epsilon u_i^{(1)} + \epsilon^{3/2}u_i^{(2)} + \cdots, \tag{11}$$
$$\phi = \epsilon\phi^{(1)} + \epsilon^{3/2}\phi^{(2)} + \cdots, \tag{12}$$

Now, substituting (5)-(12) into (1)-(4) and using the lowest order of ϵ we get as following equation :

$$u_i^{(1)} = \frac{V_p\phi^{(1)}}{V_p^2 - \sigma_i}, \tag{13}$$
$$n_i^{(1)} = \frac{\phi^{(1)}}{V_p^2 - \sigma_i}, \tag{14}$$
$$V_p^2 = \sigma_i + \frac{I_{io} + C_1}{C_1\sigma_n - C_2}, \tag{15}$$

where $C_1 = I_{io}^o\gamma_i - I_{eo}\gamma_e + I'_{no}\beta_n\alpha_1 + I''_{no}\gamma_1\alpha_1$ and $C_2 = I_{eo} + I'_{no}\beta_n\sigma_n + I''_{no}\gamma_1\sigma_n$.

Equation (15) represents the linear dispersion relation. To the next higher order of ϵ, one obtains from equations (1)-(4)

$$\frac{\partial n_i^{(1)}}{\partial\tau} - V_p\frac{\partial n_i^{(2)}}{\partial\xi} + \frac{\partial u_i^{(2)}}{\partial\xi} = 0, \tag{16}$$

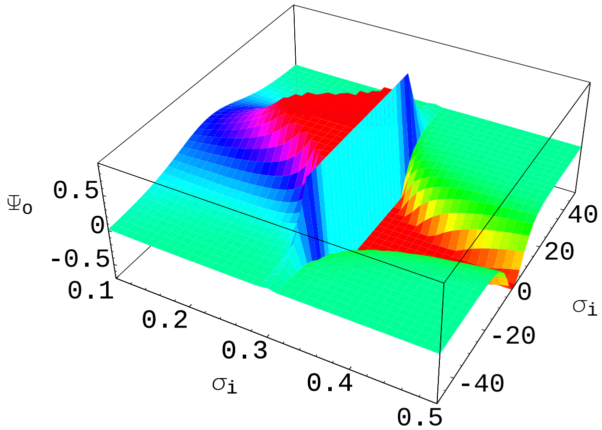

FIG. 2: (Color online) Showing the variation of Φ_0 with σ_i for solitary waves when $U_0 = 0.1$, $\sigma_n = 0.4$,and $\beta_n = 0.7$.

$$\frac{\partial u_i^{(1)}}{\partial\tau} - V_p\frac{\partial u_i^{(2)}}{\partial\xi} + \sigma_i\frac{\partial n_i^{(2)}}{\partial\xi} + \frac{\partial\phi^{(2)}}{\partial\xi} = 0, \tag{17}$$
$$\frac{\partial{\phi^{(2)}}^2}{\partial\xi} = \frac{2V_p}{(V_p^2-\sigma_i)^2}\frac{\partial\phi^{(1)}}{\partial\tau} + \left(\frac{1}{V_p^2-\sigma_i} - \sigma_n\right)\frac{\partial\phi^{(2)}}{\partial\xi}$$
$$+2(1-\beta_n){\sigma_n}^{3/2}\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}, \tag{18}$$

The combination of equations (16) and (18) gives

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0, \tag{19}$$

It is called K-dV equation where A is the nonlinear coefficient, and B is the dispersion coefficient. The nonlinear and dispersion coefficients for our present purpose read

$$A = \frac{{\sigma_n}^{3/2}}{V_p}C_1(I_{io}+C_1)(V_p^2-\sigma_i)^2(1-\beta_n), \tag{20}$$

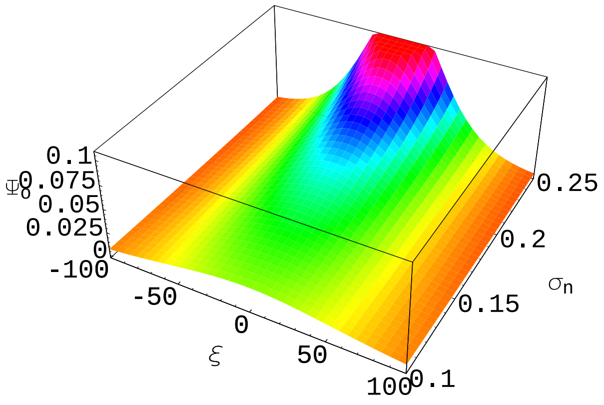

FIG. 3: (Color online) Showing the variation of Φ_0 with σ_n for solitary waves when $U_0 = 0.1$, $\sigma_i = 0.2$,and $\beta_n = 0.7$.

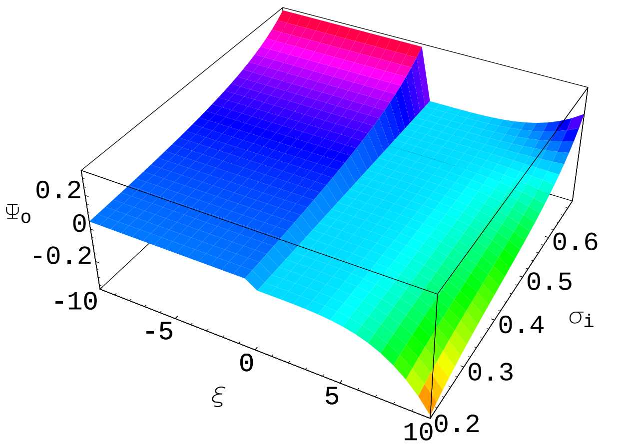

FIG. 5: (Color online) Showing the variation of Φ_0 with σ_i for shock waves when $U_0 = 0.1$, $\sigma_n = 0.4$,and $\beta_n = 0.7$.

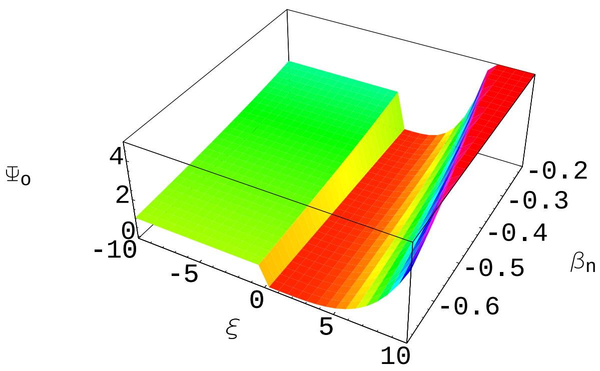

FIG. 4: (Color online) Showing the variation of Φ_0 with β_n for shock waves when $U_0 = 0.1$, $\sigma_i = 0.4$, and $\sigma_n = 0.6$.

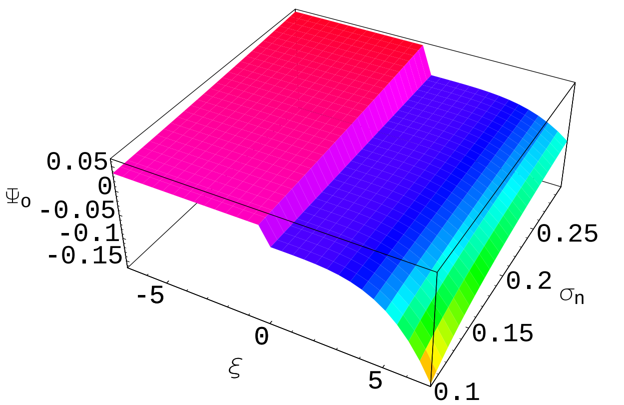

FIG. 6: (Color online) Showing the variation of Φ_0 with σ_n for shock waves when $U_0 = 0.1$, $\sigma_i = 0.2$,and $\beta_n = 0.7$.

$$B = \frac{C_1(V_p^2 - \sigma_i)^2}{2V_p I_{io}}. \tag{21}$$

IV. DERIVATION OF BURGERS' EQUATION

The reductive perturbation technique is employed to study shock profiles [19], i.e. we first introduce the stretched coordinates [18, 20]

$$\xi = \epsilon^{1/2}(x - V_p t), \tag{22}$$

$$\tau = \epsilon t, \tag{23}$$

where ϵ and V_p are the same as used in K-dV equation. Now using these new stretched coordinates (22)-(23) in (1)-(4) and the lowest order in ϵ, one can obtain the expression for $u_i^{(1)}$, $n_i^{(1)}$, and $V_p^{(1)}$ which are exactly identical with equations (13)-(15).

To the next higher order of ϵ we get the following equations:

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p\frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial u_i^{(2)}}{\partial \xi} = 0, \tag{24}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p\frac{\partial u_i^{(2)}}{\partial \xi} + \sigma_i\frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} = 0, \tag{25}$$

$$\frac{\partial {\phi^{(2)}}^2}{\partial \xi} = \frac{2V_p}{(V_p^2 - \sigma_i)^2}\frac{\partial \phi^{(1)}}{\partial \tau} + \left(\frac{1}{V_p^2 - \sigma_i} - \sigma_n\right)\frac{\partial \phi^{(2)}}{\partial \xi}$$

$$+2(1-\beta_n){\sigma_n}^{3/2}\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + \eta\frac{\partial {u_i^{(1)}}^2}{\partial \xi^2}, \tag{26}$$

The combination of equations (24) and (26) gives

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} = C\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = 0, \tag{27}$$

It is called Burgers' equation where the value of A is as before and the C is the constant for coefficient of viscosity as

$$C = \frac{\eta}{2}. \tag{28}$$

V. NUMERICAL ANALYSIS

The stationary solitary wave solutions of K-dV equation (19) is given by (under the condition that Φ is bounded at $\zeta = \pm\infty$)

$$\Phi = \Phi_0 \mathrm{sech}^2[(\zeta - U_0\tau')\Delta], \tag{29}$$

where $\zeta = \xi - U_0\tau'$ and $\tau' = \tau$, U_0 is the speed of the waves in the reference frame, $\Phi_0 = (3U_0/A)$ and $\Delta = (4B/U_0)^{1/2}$ are, respectively, the height and thickness (normalized by λ_i) of IA solitary waves moving with the speed U_0.

The stationary solutions of Burgers' equatuin (27) is given by

$$\Phi = \Phi_0 \left[1 - \tanh\frac{(\zeta - U_0\tau')}{\Delta}\right] \tag{30}$$

where $\Phi_0 = (U_0/A)$ and $\Delta = (2C/U_0)$.

The formation of both the solitary and shock structures (obtained from 29 and 30) has been investigated shown in figures 1-6, respectively. Comparing between figures 1 and 4 within the same ranges of the same parameters we have found only negative potentials for solitary wave in case of β_n, but the positive potentials are only for shock wave structures. From figures 2 and 5 we have observed both positive and negative potential in case of both solitary and shock wave structures for σ_i. But we have made comparison between figures figures 3 and 6 then an interesting point has come out as within the same ranges of the same parameters only positive potentials exist for solitary wave (obtained from 29) in case of σ_n, but both positive and negative potentials exist for shock wave structures (obtained from 30).

VI. DISCUSSION

These are interesting results of this present investigation. In the past this type analysis never made where we found positive or negative or both potential structures exit either for solitary or shock waves. But in our this theoretical analysis we have successfully observed the formation of this type of conditional potential structure using both K-dV (19) and Burgers' (27) equations and their solutions (obtained from 29 and 30).

It has been observed that the vortex-like distribution of negative ion (i. e. trapping parameter β_n) is a source of dispersion (dissipation), and is responsible for the formation of the IA solitary and shock waves. But the nonlinear wave structures modify more for the vortex-like distribution of negative ion than the normal Boltzmann distribution [12].

To conclude, the results of the present investigation should be useful for understanding the basic features of the localized ion-acoustic (IA) solitary and shock waves in space and laboratory plasmas. We, finally suggest that a laboratory experiment be performed to test the theory presented in this work.

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